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## Toward a Control Map for Niching

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### Abstract

*Niching* can allow a diverse population to cooperatively represent a single, distributed solution to the problem at hand. Successful niching mechanisms must promote both **cooperation** (i.e., co-existence of separate “species” for each desired niche), and **competition** (i.e., intensive search for the best species for each niche, and for the best niches). In this paper we seek the competitive-cooperative boundary in the space of possible niche relationships, that will allow us to successfully predict which pairs of interacting niches will survive under GA selection and which niche pairs will be resolved to yield a single winner. By combining extant models of niching equilibrium, niche maintenance, and convergence, we define the regions of cooperation and competition on a map of niching scenarios varying along the dimensions of niche overlap and relative niche fitness. We verify this predictive map of niching failure/success, and discuss its utility in allowing us to control for the competitive evolution of desired types of cooperation. Although our models are specific to the niching mechanism we call *resource sharing*, we believe the development of competitive-cooperative control maps is important for niching theory in general.

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## 1 INTRODUCTION AND BACKGROUND

In many types of genetic algorithm (GA) applications, discovering and exploiting different “niches” in the problem environment is critical. *Niching* can allow a diverse population to cooperatively represent a single, distributed solution to the problem at hand, or to present us with a set of high-quality alternative solutions. Successful niching mechanisms must promote both **cooperation** (i.e., co-existence of separate “species” for each desired niche), and **competition** (i.e., intensive search for the best species for each niche, and for the best niches).

In this paper we examine a very general, and successful, niching mechanism: the forced sharing of finite resources (or “rewards”) from the problem domain. For *resource sharing* to successfully mix cooperation and competition, the GA must be able to distinguish between cooperative and competitive pairs of species or niches. At one extreme, species whose niches have nothing in common are clearly cooperators (in a passive sense). At the other extreme, species whose niches completely overlap are clearly competitors. Somewhere between these two extremes, as niche overlap varies from zero to complete, lies the boundary between cooperation and competition, under niched GA selection. In this paper we estimate the shape and location of this boundary. For the first time we develop, and verify, a predictive map of niching. With such a map in hand, we can identify the niched GA parameters that will allow us to control for the types and levels of cooperation we want to evolve.

In the remainder of this section, we discuss niching in general before focusing on resource sharing as the niching mechanism to be analyzed.

### 1.1 THE NEED FOR NICHING

In a GA, selection drives the evolving population toward a uniform distribution of  $N$  copies of the most highly fit individual. Mutation and non-stationary fitness functions might stave off 100% convergence, but it is unarguable that the first-order effect of the first-order operator, selection, is the loss of low quality diversity. In many applications of the GA, including classifier systems, uniform convergence is undesirable. In multi-objective GA problems, for example, we might want to find a number of solutions with different tradeoffs among the multiple objectives (Horn & Nafpliotis, 1993). In the learning classifier system (LCS), we ask the GA to search through the space of all possible rules to find and maintain a diverse, cooperative subpopulation of rules that together represent a *concept*.

To prevent the best individual in the population from replacing all copies of competing rivals, some kind of *niching* (a.k.a. *speciation*) is necessary. Niching induces *restorative pressure* (Horn, 1993), to balance the *convergence pressure* of selection.

### 1.2 OTHER NICHERS

We briefly mention alternative niching methods (several of these are covered in more detail by Mahfoud (1995a; 1995b)), focusing more on sharing techniques for niching (and in particular, resource sharing, the main target of our analysis in this paper).

A number of niching mechanisms have been proposed and used over the last couple of decades. One of the earliest was Cavicchio’s *preselection* (Cavicchio, 1970; Mahfoud, 1992), in which offspring could only replace one of their parents. De Jong’s *crowding* (De Jong,

1975; Mahfoud, 1992) had the same flavor, in that new individuals replaced less-fit, but similar, solutions in the old population. Boltzmann tournament selection has also been shown to have niching effects (Goldberg, 1990; Mahfoud, 1993). Recently various kinds of *distributed population GAs*, such as "island models", have been used to find and maintain multiple solutions by allocating different subpopulations to different parts of the search space (Davidor, 1991). And explicit recognition and promotion of species is a general approach instantiated in different ways, including species tags and various clustering mechanisms.

A very successful form of niching for GAs in pure optimization tasks is *fitness sharing*. Introduced by Goldberg and Richardson (1987), studied in detail in (Deb, 1989; Horn, 1993; Mahfoud, 1995a), and challenged by a massively multimodal problem in (Goldberg, Deb, & Horn, 1992), fitness sharing works by degrading the objective (i.e., stand-alone) fitness of an individual according to the number of genotypically or phenotypically similar individuals in the current population. The resulting *shared fitness* is then used in normal GA selection and recombination. The closely related *resource sharing* induces niching implicitly (thus it has been called *implicit niching* by Horn, Goldberg, and Deb (1994)).

Resource sharing is applied to tasks in which multiple, finite, explicit resources are known and can be "attributed" or "credited" differentially to various individuals in the population. If no such explicit resources are identifiable in the problem domain, objective fitness can be considered a finite resource to be "shared", resulting in the aforementioned fitness sharing. Fitness and resource sharing are indeed so similar in their behavior that a unified model of *sharing* for niching is possible (Horn, 1997).

### 1.3 RESOURCE SHARING

A natural niching effect is implicitly induced by competition for limited resources (i.e., finite rewards). The basic algorithm common to all resource sharing systems is simple and intuitive:

1. For each of the finite resources  $r_i$ , divide it up among all qualified individuals contending for it, in proportion to their various merits (that is, the relative strengths of their claims). Thus two equally deserving individuals should be allocated equal amounts of the resource. If the resource is discrete, and cannot be evenly divided, then for each indivisible unit of the resource, randomly choose among equally deserving individuals. This random choice results in an *expected* uniform distribution of resources among equally deserving candidates.
2. For each individual, add all rewards/credits earned in the first step, and use this amount (perhaps scaled) as the fitness for GA selection.
3. After a new generation is produced, replenish/renew the resources and start over at the first step above. Continue to loop until some stopping criterion is met.

The idea of splitting up a limited resource among all competing individuals seems evident in nature and is simple to implement. Resource sharing is often incorporated in adaptive, or simulated, systems, as we illustrate below.

#### 1.4 APPLICATIONS AND INSTANCES OF RESOURCE SHARING

This simple and natural scheme has been abstracted into a number of population-based algorithms, including: learning classifier systems (LCS) (Booker, 1982; Wilson, 1987), immune system models (Smith, Forrest, & Perelson, 1993), multi-agent systems, and ecological simulations (Huberman, 1988). The resource sharing approach seems well-suited to several major, open problem domains, such as classification, general covering problems, and layout problems. Sharing attacks two major difficulties of population-based solutions to hard problems: credit assignment, and problem decomposition. Below we illustrate how sharing deals with credit assignment and dynamic sub-tasking/decomposition by examining the learning classifier system under "example sharing".

#### 1.5 THE CLASSIFIER SYSTEM AS AN EXAMPLE OF RESOURCE SHARING

An example of *resource sharing* occurs in most implementations of the *Michigan-style* learning classifier system (Horn, Goldberg, & Deb, 1994). In an LCS, the population consists of classification rules, or simply *classifiers*. These rules attempt to classify examples (either from some training set or some test set). The rules are rewarded for correct classifications of examples, earning credit for each correct classification of an example<sup>1</sup>. The sum of credits earned, over all examples, is used for each rule's fitness. This fitness is then used in normal GA selection.

In the Michigan LCS, individuals (classification rules, or *classifiers*) compete for the rewards (or credit) given for proper classification of a finite number of examples. Several researchers have shown that simply dividing up an example's reward/credit among all rules that successfully classify that example (i.e., sharing), effectively and robustly maintains a diverse set of rules that together "cover" the examples (e.g., Booker, 1982, 1989; Wilson, 1987, 1994). Thus LCS sharing is an instance of resource sharing in which the resources are the rewards/credits for the examples. Henceforth, we will consider the examples themselves to be the resources to be shared, in order to simplify our discussion. Thus this strategy is often known as *example sharing* (McCallum & Spackman, 1990; Neri & Saitta, 1995, 1996).

The sub-goal of resource sharing is to *cover* (exploit) as much of the resources as possible. The only type of interaction between individuals is competition for the same resource, and the natural mechanism for handling such competition (and encouraging search for uncovered resources) is *sharing* of contested resources. Thus similar individuals (species) share common resources by dividing them up among themselves. This simple method induces *nicing* or *speciation*, an emergent phenomenon.

The notions of competition and niche overlap are easy to visualize in the case of resource sharing. In Figure 1, the large rectangle represents the space of all *positive examples* given to the LCS for learning. The size of a circle represents the number of *examples* covered by the corresponding rule, and hence the number of correct classifications it makes. The overlaps of circles represent overlaps of coverage among rules, and thus contain the examples "shared" by two or more rules<sup>2</sup>.

To illustrate the actual sharing of resources that leads to implicit nicing: let  $f_A$  and  $f_B$  be

<sup>1</sup>And perhaps earning penalties for incorrect classifications.

<sup>2</sup>That is, the examples correctly classified by *both* rules.

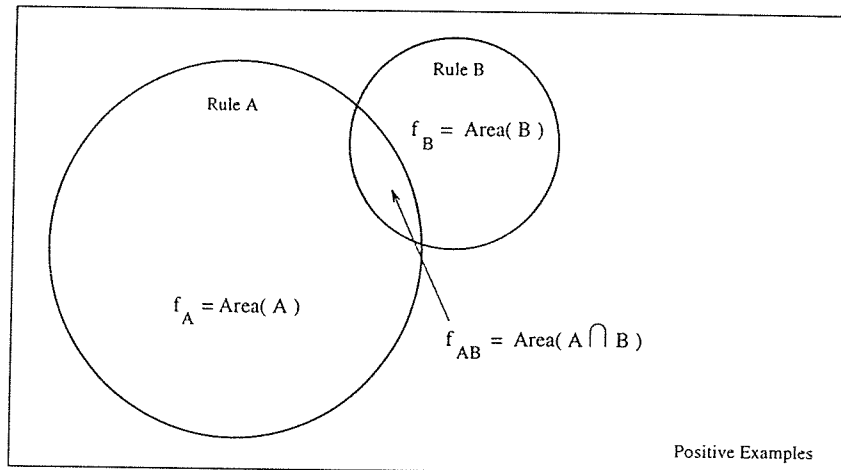


Figure 1: In the case of the learning classifier system (LCS), *implicit niching* is induced by rules competing to classify examples. We can use area in the space of examples to indicate a rule's coverage, which is also its *objective* (i.e., unshared) fitness.

the objective fitnesses for rules A and B respectively. The objective fitness could be taken as the number of examples covered<sup>3</sup> by that rule, in the case of binary classification. Let  $f_{AB}$  be the amount of resources in the overlapping coverage of rules A and B. That is,  $f_{AB}$  is the amount of resources shared by A and B (e.g., the number of examples covered by both). Let  $n_A, n_B$  be the number of copies of rules A and B, respectively, in our population of size  $N$  (thus  $N = n_A + n_B$ ). Then we can calculate the shared (expected) fitness of rule A:

$$f_{sh,A} = \frac{f_A - f_{AB}}{n_A} + \frac{f_{AB}}{n_A + n_B}. \tag{1}$$

Similarly for rule B,

$$f_{sh,B} = \frac{f_B - f_{AB}}{n_B} + \frac{f_{AB}}{n_A + n_B}. \tag{2}$$

In the next section we show how to use these shared fitness expressions to calculate the population distribution at "niching equilibrium".

### 1.6 MOTIVATION

A genetic algorithm that incorporates selection, resource sharing, crossover, and mutation, is searching for a *group* of individuals that together solve the problem at hand. In the course of that search (i.e., evolution), various groups will emerge. Some groups will be very "cooperative" in the sense that each member solves a different component (or subset of components) of the problem. Other groups will be less cooperative, with some competition among members to solve the same parts of the problem. Since this is a search problem, it is not known at the beginning of the run what kinds of cooperative and competitive groups

<sup>3</sup>In other words, classified correctly.

will be encountered. We must leave it up to selection and niching to decide which groups of interacting individuals to save and exploit, and which to break up.

We can, however, set certain parameters, such as population size or the "degree of sharing" of resources, to help selection and niching distinguish truly cooperative groups from the overly competitive ones, whenever they are discovered. The competitive groups can be dissolved, with the best of each group set free to join others in a more cooperative setting. All of this takes place automatically under niched selection. We merely want to guide the GA's decision making between cooperation and competition.

As a first step toward controlling this boundary, we attempt in this paper to develop a pre-cursor to a true control map: a *predictive* map of niching success and failure. We first need to be able to reliably predict which groups are considered cooperative and which competitive. Then we can try to control that boundary. To make this first step easier, we consider only the minimal size group, that is, two possible individuals. Even with only two possible species/niches, the difficult decision is preserved: as we increase the overlap between the two niches, at what point does the GA switch from promoting the group (i.e., cooperation) to selecting a single winner (i.e., competition)?

## 2 A REVIEW OF KEY NICHING RESULTS

In this section we restate, re-derive, and in some cases recast, some of the recently published results modeling niching under resource sharing. In particular, we revisit niching equilibrium, including its definition, maintenance and loss, and fast convergence to it, under selection. We base our later analysis on these fundamental results.

### 2.1 NICHING EQUILIBRIUM

For niched GAs that use any kind of sharing, resource or fitness, equilibrium (under selection only) will occur when the shared fitnesses of all population members are equal (Goldberg & Richardson, 1987; Deb, 1989). In the two niche case, the general equilibrium condition is:

$$f_{sh,A} = f_{sh,B}. \quad (3)$$

Substituting in the expressions for shared fitnesses above, and solving for the *equilibrium ratio*  $r_{eq,n} \equiv \frac{n_B}{n_A}$ :

$$r_{eq,n} = \frac{r'_f - r_o}{1 - r_o}, \quad (4)$$

where  $r_o$  was defined in earlier work (Horn, Deb, & Goldberg, 1994) to be the ratio of overlap:  $r_o \equiv \frac{f_{AB}}{f_A}$ . Here we define  $r'_f$  to be the inverse of the fitness ratio  $r_f$ , which was also defined in earlier work (e.g., Horn, 1993)<sup>4</sup>.

From Equation 4 we see that the equilibrium point changes with overlap, just as with fitness sharing. With no overlap, that is when  $r_o = 0$ , then  $r_{eq,n} = r'_f$ . And at complete overlap ( $r_o = r'_f$ ) the equilibrium point is 0 (i.e., the population will converge to a uniform one, consisting of all As).

<sup>4</sup>Thus,  $r'_f \equiv \frac{1}{r_f} = \frac{f_B}{f_A}$ . Note that if we assume, without loss of generality, that  $f_B \leq f_A$  (that is, niche A is always the better of the two), then  $1 \leq r_f < \infty$ , while  $1 \leq r'_f \leq 1$ . Because of their different ranges, we use both  $r_f$  and  $r'_f$  in this paper.

Let us extract from the equation for  $r_{eq,n}$  the actual *number* and *proportion* of A's in the population,  $n_A$  and  $P_A = \frac{n_A}{N}$ , which we will need later:

$$r_{eq,n} = \frac{n_B}{n_A} = \frac{N - n_A}{n_A} = \frac{1 - P_A}{P_A} = \frac{r'_f - r_o}{1 - r_o}$$

Solving for  $n_A$  and calling it  $n_{A,eq}$ :

$$n_{A,eq} = \frac{1 - r_o}{1 - 2r_o + r'_f} N \tag{5}$$

This is the number of copies of A we would expect to find at equilibrium, given a particular ratio of fitness  $r'_f$  and ratio of overlap  $r_o$ , and a fixed population size  $N$ . Dividing both sides by  $N$  gives  $P_A$ , which we'll call  $P_{A,eq}$ :

$$P_{A,eq} = \frac{1 - r_o}{1 - 2r_o + r'_f} \tag{6}$$

This is the *proportion* of As we would expect to find at equilibrium (and is independent of population size  $N$ ). Both  $n_{A,eq}$  and  $P_{A,eq} = \frac{n_{A,eq}}{N}$  are simply *predictions* describing the expected equilibrium based solely on the assumed condition that at equilibrium all (shared) fitnesses of all individuals are equal.

## 2.2 EXPECTED NICHE MAINTENANCE TIMES

How many generations can we expect selection to keep both of our niches in the population? If we assume that there is no mechanism for reintroducing individuals, our question becomes "when do we expect to lose the last individual from a niche?" In the two niche case, this will happen when one species/niche has completely taken over the population.

Since niching "pushes" the population toward equilibrium, we should expect that the time to niche loss will be long. Indeed, under perfect sharing, with no niche overlap, we can calculate the expected *niche maintenance time* exactly, and we find that it grows exponentially in population size  $N$  (Horn, 1993; Mahfoud, 1995a).

For the case of niche overlap however, an exact calculation appears impossible, and some have resorted to full Markov chain models (e.g., Horn, Goldberg, & Deb, 1994), to calculate the expected absorption times. But it is desirable to have in hand a closed form expression for the expected time to niche extinction. Horn and Goldberg (1996) approximated the expected time to niche loss as

$$E[t_{abs}] = \frac{(1 - 2r_o + r'_f)^N}{(1 - r_o)^N + (r'_f - r_o)^N} \tag{7}$$

Their approximation seems an adequate predictor, as shown in Figure 2, which tests the closed-form approximation. Figure 2 plots both the exact results (from the Markov chain model) and the approximations. The solid dots are the plotted points for the exact (Markov) model, while the dashed lines are defined by Equation 7. In general, we see close agreement over the range plotted, except for high overlap (e.g., complete overlap when  $r_o = r'_f = 0.5$ ).

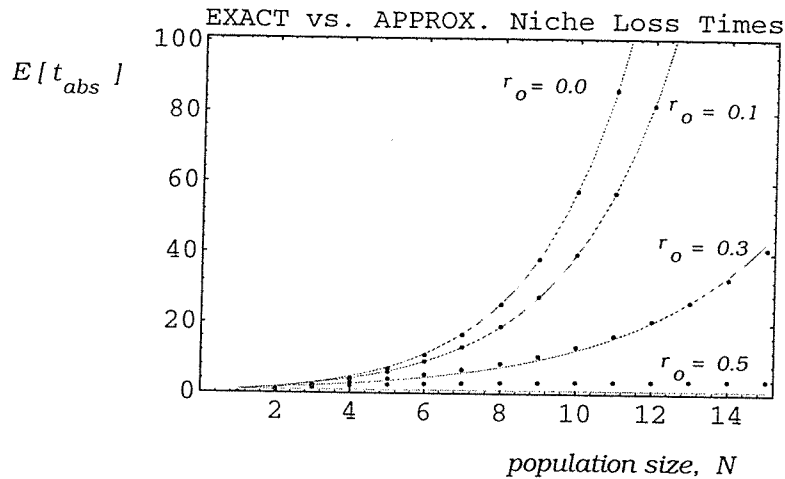


Figure 2: A comparison of *exact* expected niche loss times to the approximated times, as a function of population size. The exact results (from the Markov models) are shown as solid dots. The approximations, from the closed-form expression, are shown as dashed lines. The plots indicate general agreement for small niche overlap  $r_o$ . For all plots shown  $r'_f = \frac{1}{2}$ .

### 2.3 CONVERGENCE TO NICHING EQUILIBRIUM

Having found an upper bound on expected lifetimes of niches, we now turn to the question of how such steady-states are reached. Under normal selection (e.g., in a simple GA) it has been shown that convergence to the "equilibrium point" of a uniform population takes place very quickly. In the case of selective preference (i.e.,  $r'_f \neq 1$ ) the convergence time grows logarithmically in population size  $N$ , while under genetic drift (i.e.,  $r'_f = 1$ ), the expected convergence time grows linearly in  $N$  (Goldberg & Deb, 1991). Can we expect similarly quick convergence to "niching equilibrium"?

So called "niche convergence time" is also of interest because it provides a *lower bound* on niche maintenance time. We can compare this lower bound with our upper bound on niche maintenance times. When these two time bounds are close, we can expect poor niche maintenance. When they are far apart, we should expect long-term, steady-state niche maintenance. Under what conditions of niche overlap, fitness ratio, and population size can we expect to find the *phase transition* from poor to good niche maintenance; that is, the transition from competition to cooperation?

#### 2.3.1 Expected Proportions Analysis

To answer such questions, we use the simple, well-known method of *expected proportion equations* to model niche convergence. This method of tracking the expected next generation *population* only, rather than tracking the entire *distribution* over all possible populations, has been put to good use many times in the GA literature (e.g., Smith & Valenzuela-Rendón, 1989; Goldberg & Deb, 1991; Neri & Saitta, 1995).

Under proportionate selection, the expected proportion of the next generation's population



given to an individual is *equal* to the probability of selecting that individual for reproduction in the current generation's population. Thus if  $P_{A,t}$  is the proportion of the current population, at time  $t$ , consisting of copies of rule **A**, then  $E[P_{A,t+1}] = p_A$ , where  $p_A$  is calculated at time (generation)  $t$ . The probability  $p_A$  of selecting an **A** can be written in terms of the ratio of niche overlap  $r_o$ , the inverse ratio of fitness  $r'_f$ , and of course the current proportion of **A**s ( $P_{A,t}$ ):

$$p_A = \frac{1 - r_o + r_o P_{A,t}}{1 - r_o + r'_f} \tag{8}$$

Substituting  $p_A$  into  $E[P_{A,t+1}] = p_A$  and rearranging, we find that

$$E[P_{A,t+1}] = \frac{1 - r_o}{1 - r_o + r'_f} + \frac{r_o}{1 - r_o + r'_f} P_{A,t}$$

Now we make our major assumption that  $E[P_{A,t+1}] \approx P_{A,t+1}$  and the above becomes a simple linear recurrence relation on  $P_{A,t}$ . Defining  $P_{A,0}$  to be the initial population, we solve the difference equation and rearrange to get:

$$P_{A,t} = \frac{1 - r_o}{1 - 2r_o + r'_f} + \left( P_{A,0} - \frac{1 - r_o}{1 - 2r_o + r'_f} \right) \left( \frac{r_o}{r'_f - r_o + 1} \right)^t \tag{9}$$

Remembering that at equilibrium (steady-state)  $P_{A,eq} = \frac{1 - r_o}{1 - 2r_o + r'_f}$  (Equation 6), and introducing  $\beta \equiv \frac{r_o}{r'_f - r_o + 1}$ , we simplify to

$$P_{A,t} = P_{A,eq} - (P_{A,eq} - P_{A,0})\beta^t$$

or

$$P_{A,t} = P_{A,eq}(1 - \beta^t) + P_{A,0}\beta^t \tag{10}$$

We note that  $\beta \leq 1$ . This must be so as the numerator of  $\beta$ ,  $r_o \equiv \frac{f_{AB}}{f_A}$ , must always be  $\leq 1$ , while the denominator must always be  $\geq 1$ , since  $f_{AB} \leq f_B \Rightarrow \frac{f_{AB}}{f_A} \leq \frac{f_B}{f_A} \Rightarrow r_o \leq r'_f$ , and so  $1 \leq r'_f - r_o + 1$ . In general,  $\beta < 1$  and Equation 10 illustrates the exponential decay of the initial state's effect on the current population, and the corresponding exponential growth of the "long term" steady state's influence.

### 2.3.2 Convergence Times

Here we derive a useful expression for two-niche convergence times by solving Equation 10 for time  $t$  (in generations). Rearranging Equation 10, we move  $t$  to one side:

$$\beta^t = \frac{P_{A,t} - P_{A,eq}}{P_{A,0} - P_{A,eq}}$$

Taking the logarithm of both sides and solving for  $t$ , yields:

$$t = \frac{\ln\left(\frac{P_{A,t} - P_{A,eq}}{P_{A,0} - P_{A,eq}}\right)}{\ln \beta} \tag{11}$$

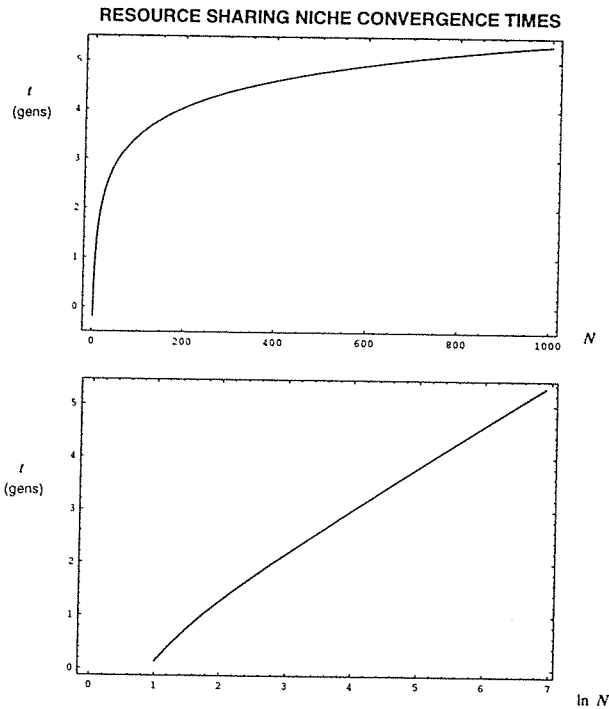


Figure 3: Expected niche convergence time grows logarithmically in population size  $N$ .

In general  $P_{A,t}$  approaches  $P_{A,eq}$  asymptotically with  $t$ . More practically, we introduce the discrete nature of a finite population of size  $N$ , by asking for the time it takes to go from one copy of **A** ( $P_{A,0} = \frac{1}{N}$ ) to within at most one individual ( $\frac{1}{N}$ ) of the equilibrium population ( $P_{A,t} = P_{A,eq} - \frac{1}{N}$ ):

$$t = \frac{\ln\left(\frac{\frac{1}{N}}{P_{A,eq} - \frac{1}{N}}\right)}{\ln \beta}$$

Simplifying yields:

$$t_{conv} = \frac{-\ln(P_{A,eq}N - 1)}{\ln \beta} \tag{12}$$

We can see immediately that expected convergence time grows logarithmically in population size  $N$ , as we suspected. Figure 3 illustrates a typical expected growth in convergence time with increasing  $N$ . Here  $r'_f = \frac{1}{2}$  and  $r_o = \frac{1}{3}$  (remember that  $\beta$  is a function of  $r'_f$  and  $r_o$ ).

### 3 A MAP OF COOPERATION VERSUS COMPETITION

In our review of previous work, we have shown that convergence to niching equilibrium is fast, behaving much like convergence in the simple GA under selection pressure alone. Both convergence times, for the simple GA and for sharing, grow logarithmically in  $N$ ,

population size. This is in sharp contrast to the very long niche maintenance times, which grow very rapidly (exponentially) in  $N$ . But with both niche maintenance and niche convergence, performance degrades with increasing niche overlap, objective fitness difference, and decreasing population size. As the general niching conditions become less favorable, the difference between niche convergence time and expected niche loss (maintenance) time narrows. We face a blurry area between cooperating species (i.e., long niche maintenance times) and competing species (e.g., very short maintenance times).

In this section we use the previous results to further define the elusive but critical boundary between competition and cooperation. We develop a preliminary version of a "control map" to predict the bounds of niching success and failure, comparing theoretical boundaries with empirical results, and discuss the implications and applications of such control maps.

### 3.1 COMPARING NICHE MAINTENANCE TIMES TO NICHE CONVERGENCE TIMES

In this subsection we make an initial attempt to bring the above two timing results together. In Figure 4 we illustrate a simple way of using these two results together. The upper curve plots the expected niche extinction times (using the approximations) as a function of population size. The lower curve is the expected niche convergence time. Here again the fitness ratio  $r'_f = \frac{1}{2}$ , but now the overlap ratio is relatively high,  $r_o = 0.45$ . This overlap means that  $r_o/r'_f = 90\%$  of B's covered examples are also covered by A. This relatively high ratio of overlap was chosen to illustrate clearly the predicted niching failure at low population sizes (e.g.,  $N < 20$ ). At high enough population sizes (e.g.,  $30 < N$ ), the difference between the niche convergence and niche extinction times is quite high.

Characterizing the critical phase transition from poor niche maintenance to robust niche maintenance as population size increases, is tricky. The niche convergence model (lower curve) breaks down for small population sizes, while the niche extinction time approximation (upper curve) loses accuracy with increasing overlap. Yet our initial models above give us clear indication that sharing works well *within bounds*. And apparently we have some hope of defining those boundaries. In the following subsections we attempt to find these boundaries.

### 3.2 EQUATING NICHE MAINTENANCE AND CONVERGENCE TIMES

In Figure 4 we saw that small population sizes  $N$  could bring together niching failure time  $t_{abs}$  and niche convergence time  $t_{conv}$ , implying niching failure and hence inter-niche competition. On the other hand, large enough  $N$  could mean fast convergence and long times to failure, which in turn implies successful niching and hence cooperation. Perhaps if we equate  $t_{abs}$  and  $t_{conv}$ , we can find a meaningful boundary between cooperative pairs of niches and competitive pairs.

To put it in more concrete terms, note that  $t_{conv}$  and  $t_{abs}$  are functions of overlap, fitness ratio, and population size. Thus  $t_{conv}(r_o, r_f, N)$  and  $t_{abs}(r_o, r_f, N)$  denote functions. The region defined by  $t_{conv} = t_{abs}$  is a surface through the three dimensional space  $\langle r_o, r_f, N \rangle$ . To aid visualization, we fix  $N$  and get a two-dimensional space with  $t_{conv} = t_{abs}$  now defining a line. On one side of that line,  $t_{conv} > t_{abs}$ , and niching fails, while on the other side  $t_{conv} \ll t_{abs}$  and niching succeeds.

With all of the noise in the stochastic operators of a GA, expecting a crisp line is unrealistic.

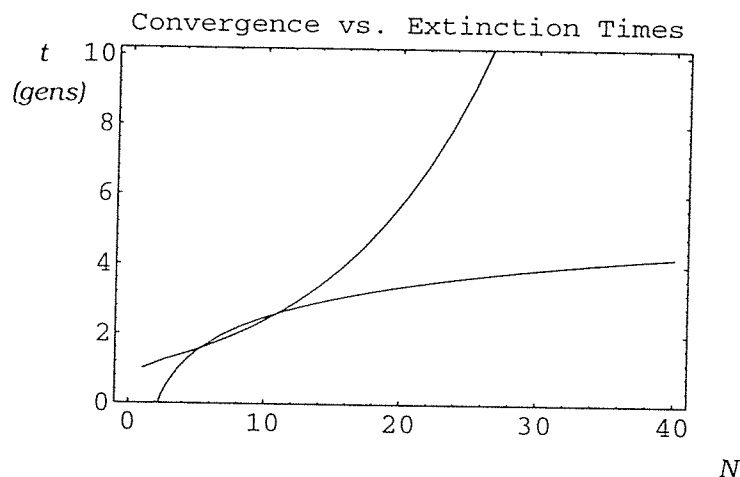


Figure 4: Expected niche extinction times (upper curve) versus expected niche convergence times (lower curve). Here fitness ratio  $r_f = 2$  with very high overlap  $r_o = 0.45$  (near maximum).

But we could easily define a boundary *region* as the space between two crisp bounds. One bound might be defined as  $c_{succ}t_{conv} = t_{abs}$ , where  $c_{succ} > 1$  is some constant representing how much longer than convergence time  $t_{conv}$  we expect niche maintenance time  $t_{abs}$  to be, in order to predict niching success. Thus increasing  $c_{succ}$  raises our requirement for cooperation. Niche maintenance times  $t_{abs}$  for successfully cooperating species would have to be at least  $c_{succ}$  times their convergence times  $t_{conv}$ .

Similarly we might define another bound as  $c_{fail}t_{conv} = t_{abs}$  (where  $c_{fail}$  might be  $< 1$ ). Thus for two niches to be considered truly competing, such that we can count on a quick resolution to their conflict and a single emergent winner, their expected niche maintenance time must be no more than  $c_{fail}$  times longer than their expected convergence time.

Between the two upper and lower boundary lines would be a gray area of niching scenarios, in which we could not predict with much confidence if both niches would be maintained, or if a single clear winner would emerge. But above the boundary region we could say with some confidence (and perhaps a quantifiable amount) that such niche pairs are cooperative and would be maintained, while below the boundary region we could say with confidence that niches in such pairs are competitive, and that only one of each pair will win and be represented in the population.

Figure 5 gives a hypothetical example. Here each possible niching situation is a coordinate in the space  $\langle r_o, r_f \rangle$ , since we have fixed  $N$  to some value. We would like to be able to divide the space into cooperative and competitive regions. We could then adjust GA and niching parameters, such as the exponent of fitness scaling, or some tolerance of niche overlap (e.g.,  $\sigma_{sh}$  or  $\alpha_{sh}$ ), or population size  $N$ , etc., to get the cooperative-competitive boundary that we want. We could then say, "yes, pair  $X$  should be considered cooperative; I'd want them both. But pair  $Y$  should be treated as a competitive relationship; I want only the better species of that pair." (See Figure 5.)

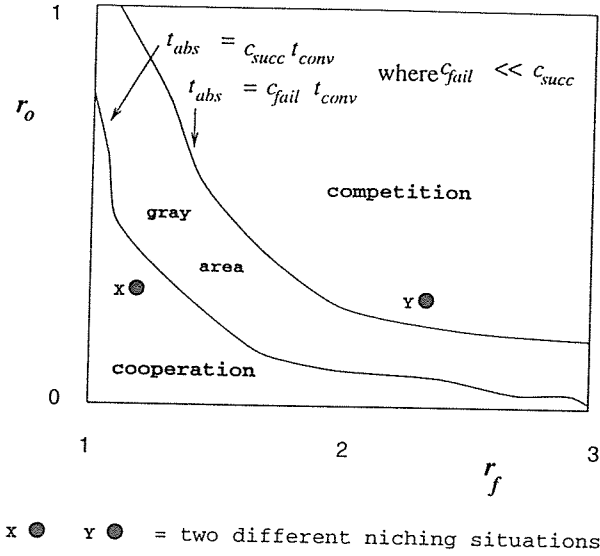


Figure 5: Speculative cooperative-competitive boundary for resource sharing given population size  $N$ , found by setting  $c t_{conv} = t_{abs}$ .

### 3.3 AN EXAMPLE: A CONTROL MAP FOR RESOURCE SHARING

In this subsection we develop theoretical bounds for niching under resource sharing by solving the equation  $c t_{conv} = t_{abs}$ , and then using large and small values of  $c$  for success and failure bounds respectively. Retrieving  $t_{abs}$  from Equation 7, and  $t_{conv}$  from Equation 12:

$$c t_{conv} = t_{abs},$$

$$c \frac{-\ln(P_{A,eq} N - 1)}{\ln \beta} = \frac{(1 - 2r_o + r'_f)^N}{(1 - r_o)^N + (r'_f - r_o)^N},$$

$$c \frac{-\ln(\frac{1-r_o}{1-2r_o+r'_f} N - 1)}{\ln(\frac{r_o}{r'_f-r_o+1})} = \frac{(1 - 2r_o + r'_f)^N}{(1 - r_o)^N + (r'_f - r_o)^N}. \tag{13}$$

Before proceeding to solve the above equation (for  $r_o$  as a function of  $r_f$  and  $N$  for example), we pause to simplify the notation. Recall that we defined the ratio of overlap  $r_o$  as  $r_o \equiv \frac{f_{AB}}{f_A}$ ; that is, the ratio of the amount of overlap,  $f_{AB}$ , to the fitness  $f_A$  of the better niche A. But since A is the better niche, and hence  $f_A \geq f_B$ , then  $r_o$  must always vary between 0 and  $r'_f$ , since  $f_{AB}$  reaches its maximum at  $f_{AB} = f_B$ , as we noted earlier. Thus the upper limit ratio  $r_o$  varies with the ratio of fitness  $r_f (= \frac{f_A}{f_B})$ . For the sake of our analysis, however, it is desirable to have the ratio of overlap vary from 0 to 1 always, no matter what  $r_f$  is. This will allow for easier algebraic manipulation and simplified visualization. We therefore introduce another ratio of overlap:  $r_{ov} \equiv \frac{f_{AB}}{f_B}$ . Note that  $r_{ov}$  varies from 0 to 1. Also note that  $r_{ov} = \frac{f_{AB}}{f_B} = \frac{f_{AB}}{f_A} \frac{f_A}{f_B} = r_o r_f$ . Thus  $r_o = \frac{r_{ov}}{r_f}$ , and we can simply substitute  $\frac{r_{ov}}{r_f}$  for  $r_o$  in

Equation 13 above:

$$c \frac{-\ln\left(\frac{r_f - r_{ov}}{r_f - 2r_{ov} + 1} N - 1\right)}{\ln\left(\frac{r_{ov}}{1 - r_{ov} + r_f}\right)} = \frac{(r_f - 2r_{ov} + 1)^N}{(r_f - r_{ov})^N + (1 - r_{ov})^N}. \quad (14)$$

Solving Equation 14 for a closed-form expression is difficult. Even after fixing population size  $N$ , we would still find it difficult to solve the above for  $r_{ov}$  as a function of  $r_f$ , or for  $r_f$  as a function of  $r_{ov}$ . With one side of the equation being logarithmic and the other being polynomial, it is not straightforward to solve such equations for closed-form expressions. We could turn to transcendental functions or approximations for closed-form solutions. For now, we use numerical solutions obtained via a computer software package for mathematics (namely, Mathematica<sup>5</sup> and its "FindRoot" function). That is, given a specific population size  $N$  and fitness ratio  $r_f$ , we numerically solve for  $r_{ov}$  (using Mathematica's "FindRoot" and giving it a starting point of  $r_{ov} = 0.9999$ ).

Numerically solving Equation 14 for  $r_{ov}$ , over a series of  $r_f$  values and a fixed  $N$ , we can interpolate a bound (for a given  $c$ ). For example, if we assume a population size  $N = 50$ , a constant  $c = 10$ , and then vary  $r_f$  from 1 to 4, we can plot  $r_{ov}$  as a function of  $r_f$  (by sampling  $r_f$  at intervals of 0.1 and then interpolating), to yield the upper plot in Figure 6. Similarly, changing  $c$  to 1000, we get the lower plot in Figure 6. Notice that as  $c$  increased, the boundary decreased. Intuitively, the greater the difference  $c$  we are asking for between convergence and maintenance times, the less overlap and fitness difference we can tolerate. Thus the area above the  $c = 10$  plot should represent failed niching, while the area below the  $c = 1000$  line represents successful niching (surviving pairs of niches)<sup>6</sup>. The area in between the two bounds is the "gray area" in which we cannot reliably predict success or failure of niching.

Thus Figure 6 is the predictive success map for pairs of niches, covering the range of possible scenarios (fitness versus overlap). Next we perform the experiments to see how well theory corresponds to reality.

### 3.4 EMPIRICAL RESULTS

Rather than perform actual runs of a GA with resource sharing among two overlapped niches, we simply use the Markov chain model from (Horn, Goldberg, & Deb, 1994). Since this model is exact, it gives us the expected results for an arbitrarily large number of runs of a real GA. In particular, we use the Markov chain for resource sharing to give us the *expected niching performance* for two niches with  $N = 50$ , fitness ratios  $r_f$  varying from 1 to 4, and overlap varying from none ( $r_{ov} = 0$ ) to complete ( $r_{ov} = 1$ ), below.

We define *expected niching performance* as the probability that both niches are still represented in the population at some arbitrary point in the future. The states of the finite Markov chain for two niches correspond to the possible distributions of the size  $N$  population between the two niches (Horn, Goldberg, & Deb, 1994). Exactly two states correspond to uniform populations (i.e., one for all As and one for all Bs). These states are considered *absorbing*; once the Markov process (i.e., the GA run) enters such a state, it can never leave

<sup>5</sup>From Wolfram Research, Inc., Champaign, IL, USA.

<sup>6</sup>The values of  $c$ , namely 10 and 1000, are chosen arbitrarily. This limitation is discussed later in this paper.

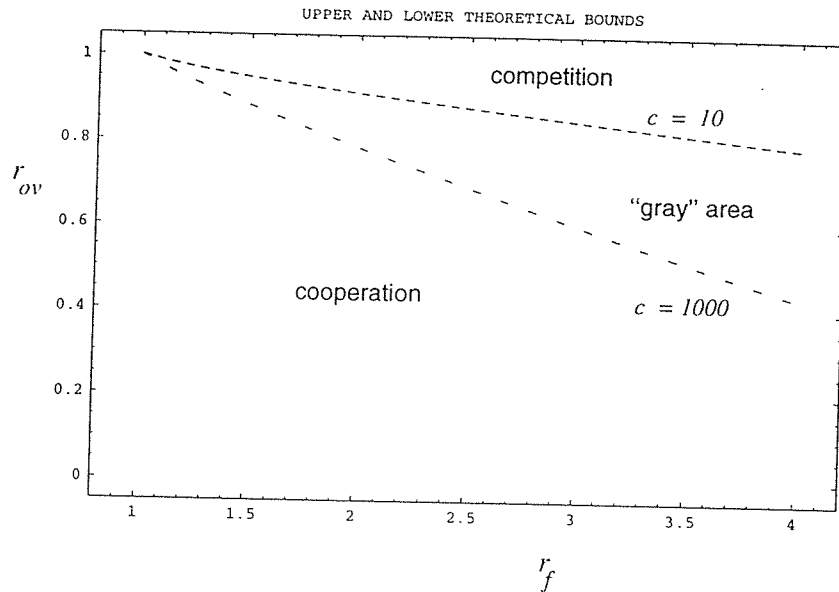


Figure 6: Theoretical *cooperative-competitive* boundary for resource sharing given population size  $N = 50$ , and by arbitrarily choosing  $c = 10$  for the niching failure boundary (the lower bound on competition) and  $c = 1000$  for the niching success boundary (upper bound on cooperation).

it. (We are assuming no mutation, so whenever all the representatives of a niche are lost from the population, the niche is lost forever.)

Recall that we can use the Markov chain directly to calculate the probability that the population is in a given distribution (i.e., that the chain is in a given state) after any number of generations. We can therefore ask, given a particular initial distribution at generation  $t = 0$ , what is the probability that the process has *not* been absorbed into one of the states corresponding to a uniform population (of all one niche), after some number  $t$  generations. This probability, which is one minus the probability of being absorbed by generation  $t$ , is the probability of niching success (that is, both niches have survived). Since the Markov model is an exact model of a niched GA, the calculated probability is equivalent to the expected number of GA runs, using different random number seeds, in which both niches are present at generation  $t$ .

For our “empirical results” we fix population size  $N$  to 50 individuals, and choose  $t = 200$  generations as a somewhat arbitrary but long period for niches to survive. We choose as an initial population distribution the expected distribution corresponding to a random initialization<sup>7</sup>. Computing the probability of both niches surviving 200 generations requires the multiplication of the initial population distribution vector with the Markov transition matrix 200 times (once for each generation), and then adding the probabilities for the two absorbing states (corresponding to all As and all Bs) and subtracting this sum from one<sup>8</sup>.

<sup>7</sup>Since there are only two niches A and B, and hence only two possible individuals, the distribution is a binomial one, over the spectrum from “all Bs” to “all As”.

<sup>8</sup>Or, equivalently, adding up the probabilities for all of the transient states.

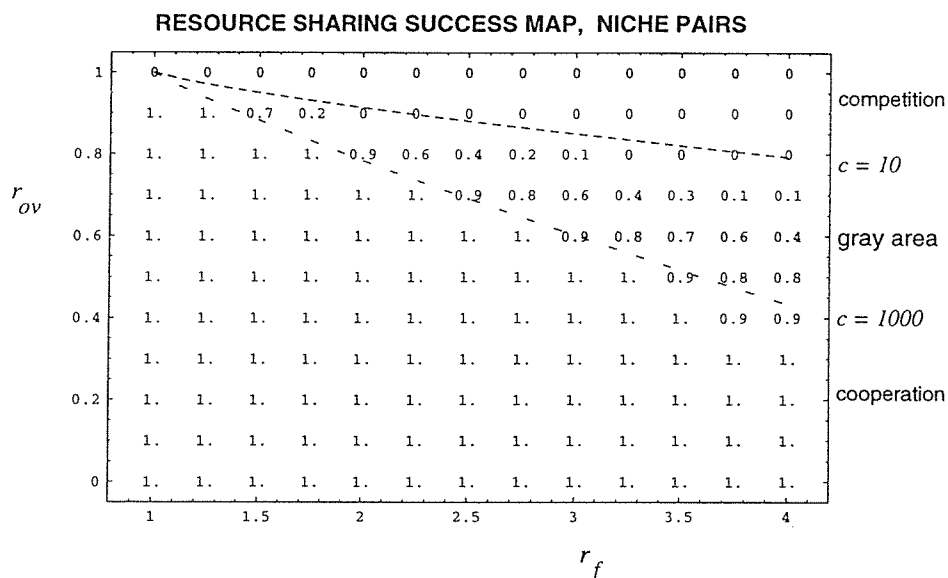


Figure 7: Analytical results superimposed on empirical results: the numbers plotted are the expected survival rate for the niche *pairs* (i.e., both niches survive), after  $t = 200$  generations. These niching success probabilities are obtained to infinite precision via the Markov chain, but are here shown rounded to the nearest tenth.

In Figure 7 we show these probabilities, rounded to the nearest tenth<sup>9</sup>, for various degrees of niche overlap and fitness ratios. We can call these probabilities *niche performance*, or “niche survival rate”. Also shown in Figure 7 are the theory bounds from Figure 6.

## 4 DISCUSSION

The results in Figure 7 reveal some important issues and hide some others. These are issues that bear analysis, including the seemingly close agreement between the model’s predictions and the actual results, the almost linear nature of the boundaries, and the question: “what is all of this good for?”. These and other points are discussed below, in terms of limitations, contributions, and future work.

### 4.1 LIMITATION ON PREDICTIVE ABILITY

The analytical boundaries and the actual expected performance data seem to agree quite well, but such agreement must be qualified. We have seen that the analytical bounds are sensitive to the setting of the constant  $c$ , as they should be, but we have no theory to guide us in choosing the appropriate  $c$  values for the success and failure boundaries. Instead, we chose  $c$  values of 10 and 1000 to approximately fit the data “after the fact”. Now it is true that the data do bear out our theoretical expectations that the general equation  $c t_{conv} = t_{abs}$  could be used to find the boundaries of niching success and failure. And it is

<sup>9</sup>I.e., one significant digit.



also true that our prediction that the constant for the competition bound should be much smaller (by several orders of magnitude) than the constant for the cooperation bound (i.e.,  $c_{fail} \ll c_{succ}$ ). But it is clear that without some theory for setting  $c$ , we cannot fully use our models to *predict* niching success/failure. However, we believe that future work can produce guidance for setting  $c_{succ}$  and  $c_{fail}$  based on analysis of the relative effects of noise from nondeterministic sources (e.g., selection, crossover, sharing operators).

Future work should also more closely examine the effect of  $t$  on the boundaries; that is, how the niche failure and success boundaries change over the generations. Note that our theoretical model does not include the number of generations  $t$  as a parameter, but of course our experimental results must. Our models therefore implicitly "predict" that the boundaries do not change (much) over a large range of  $t$ . Although Figure 7 shows only the results for  $t = 200$ , a number of other ending generations were looked at, including  $t = 50, 100, 150, 250,$  and  $300$ . All had similar niching success rates as shown in Figure 7, but with slightly decreasing probabilities as  $t$  increased (as expected). Thus we are claiming here that the effect of  $t$  on niching success/failure is not nearly as significant as the effects of  $N, r_f,$  and  $r_{ov}$ , but more analysis and experiment is needed to support this conjecture.

#### 4.2 A QUASI-LINEAR RELATIONSHIP

Figure 7 indicates that both analysis and experiment agree on the nearly linear relationship of  $r_{ov}$  to  $r_f$  along the contours of constant probability of niching success. But Equation 14, the source of our theoretical bounds, is highly non-linear. A closer analysis (below) of Equation 14 suggests a dominant linear term. First, we repeat Equation 14:

$$c \frac{-\ln\left(\frac{r_f - r_{ov}}{r_f - 2r_{ov} + 1} N - 1\right)}{\ln\left(\frac{r_{ov}}{1 - r_{ov} + r_f}\right)} = \frac{(r_f - 2r_{ov} + 1)^N}{(r_f - r_{ov})^N + (1 - r_{ov})^N}$$

We note that for  $r_f \approx 1$ , the right hand side essentially reduces to

$$\frac{(r_f - 2r_{ov} + 1)^N}{(r_f - r_{ov})^N + (r_f - r_{ov})^N} \Rightarrow \frac{(r_f - 2r_{ov} + 1)^N}{2(r_f - r_{ov})^N} \Rightarrow \frac{1}{2} \left(\frac{r_f - 2r_{ov} + 1}{r_f - r_{ov}}\right)^N$$

Writing the full equation, and rearranging slightly, yields

$$\sqrt[N]{2c \frac{-\ln\left(\frac{r_f - r_{ov}}{r_f - 2r_{ov} + 1} N - 1\right)}{\ln\left(\frac{r_{ov}}{1 - r_{ov} + r_f}\right)}} = \frac{r_f - 2r_{ov} + 1}{r_f - r_{ov}}$$

Calling the left hand side  $K$ , with  $K \equiv \sqrt[N]{2c t_{conv}}$ , we rewrite the above as

$$K = \frac{r_f - 2r_{ov} + 1}{r_f - r_{ov}},$$

and solve for  $r_{ov}$ :

$$r_{ov} = \left(\frac{1 - K}{2 - K}\right) r_f + \frac{1}{2 - K},$$

which is linear in  $r_f$  if  $K$  is constant. Just how much does  $K$  vary? From the definition of  $K$  we see that for large population sizes  $N$  we are dealing with a very small root of  $2c t_{conv}$ . If  $t_{conv}$  does not vary much with  $r_f$ ,  $K$  can be considered more or less constant<sup>10</sup>.

<sup>10</sup>Note that  $t_{conv}$  is logarithmic in  $r_f$ .

In future work we might investigate the dominant terms in Equation 14 and look for an approximation that yields a compact, closed-form expression relating  $r_{ov}$  to  $r_f$  linearly, as a function of  $c$  and  $N$ .

### 4.3 WHAT WE HAVE NOW VERSUS WHAT WE HAD BEFORE

A natural question to ask after our comparison of empirical and theoretical niching bounds is: what were our previous theoretical bounds and how do they compare? Prior to the current analysis, the only quantified bounds to be found in the literature are the simple ones that come from the equilibrium equations directly. For example, we recall the equilibrium condition for resource sharing, Equation 6:

$$P_{A,eq} = \frac{n_A}{N} = \frac{1 - r_o}{1 - 2r_o + r_f},$$

where  $n_A$  is the number of copies of species A.

Certainly, we can expect difficulty maintaining both niches if the expected number of copies of B at equilibrium is less than one:  $n_B < 1$ . (Recall that we have all along assumed that B is the lesser fit niche:  $f_B \leq f_A$ .) So we look for the boundary  $n_B = 1$ . Substituting  $n_B = N - n_A = 1 \Rightarrow n_A = N - 1$  into the equation above, and also substituting  $r_o = \frac{r_{ov}}{r_f}$  to put the bound in terms of our new overlap ratio, and rearranging, we get:

$$r_{ov} = \frac{N - r_f - 1}{N - 2}. \quad (15)$$

This bound is then our “old bound on competition”, and is shown as the upper solid line in Figure 8. Using only our equilibrium condition, we could at least safely say that for  $r_f$ ,  $r_{ov}$  situations above this line, we can expect niching failure (and hence pure competition), since  $< 1$  copies of the lesser niche are expected to survive at equilibrium.

As for an upper bound on cooperation, below which we can be sure both niches will be maintained, we really did not have a rigorous boundary. Without models of niching performance under overlap, it was often assumed that niching would fail if any overlap existed<sup>11</sup>. While much empirical evidence of niching’s robustness existed (e.g., Deb, 1989; Goldberg, Deb, & Horn, 1992), and even some theoretical work indicated graceful degradation of niching performance with increasing niche overlap (e.g., Horn, 1993), no measure of sharing’s tolerance of overlap has been proposed. Thus we indicate the “old bound on cooperation” in Figure 8 as the flat line just at or just above  $r_{ov} = 0$ .

Between the two solid lines indicating the old bounds on competition and cooperation is the large “gray area” indicating situations of fitness and overlap in which we had no theoretical guidance for predicting niching success or failure. Figure 8 also shows our new bounds developed in this paper, for comparison. The newer bounds seem to more tightly bound the *actual* gray area of niching.

<sup>11</sup>Hence some of the criticisms of sharing as “brittle”, as well as the frequent assumptions of “perfect discrimination” (e.g., Mahfoud, 1995a).

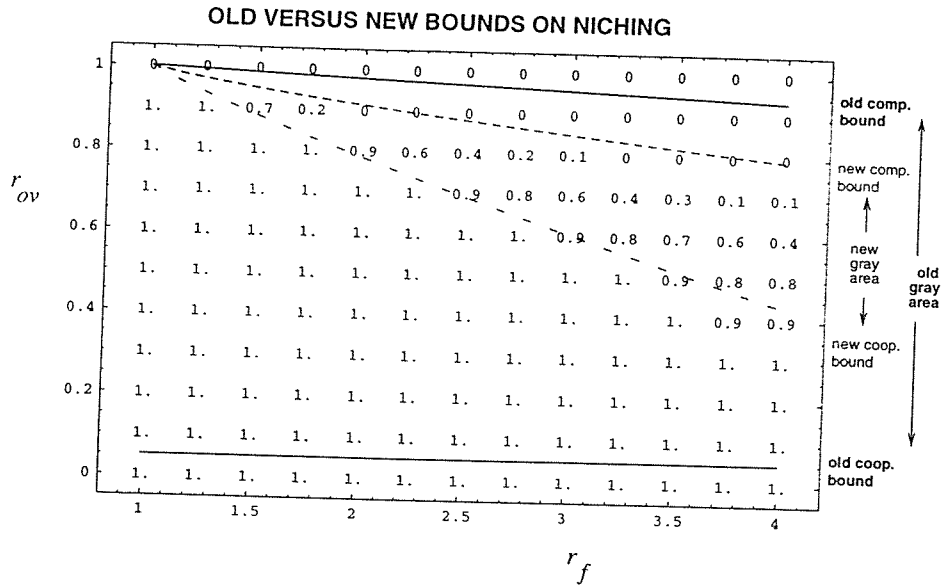


Figure 8: A comparison of our previous theoretical predictions (solid lines) of pair-wise niching success and failure versus our new bounds (dashed lines), both superimposed on the actual results.

#### 4.4 THE COMPLETE PICTURE: ALL POSSIBLE TWO-NICHE SCENARIOS

We can extend our visual analysis in Figure 8 to include all possible niching scenarios by considering the inverse fitness ratio defined earlier (as  $r'_f = \frac{1}{r_f} = \frac{f_B}{f_A}$ ). Since we assume  $f_B \leq f_A$ , then  $r'_f$  varies from 1 and approaches 0, as  $f_A$  increases from  $f_B$  towards  $\infty$  (or, equivalently, as  $f_B$  decreases from  $f_A$  to 0).

Substituting  $\frac{1}{r'_f}$  for  $r_f$  into Equations 14 and 15, we "re-plot" Figure 8 as Figure 9, including all four old and new, competitive and cooperative bounds. We also compute new experimental results (via the Markov chain model) for niching scenarios out to  $r'_f = 0$ . (Recall that for all of these results,  $N = 50$ .)

As Figure 9 shows, the new bounds remain superior to the old bounds. One may perceive, however, decreasing accuracy of the new models at the extreme of fitness difference between niches. Future work should investigate this trend, but we have already noted that some of the assumptions in our models break down for large fitness difference or extreme overlap.

#### 4.5 LIMITATION: PREDICTION WITHOUT CONTROL

Although Figures 7 through 9 seem fairly accurate, they are not as directly useful as the mixing control maps of (Thierens, 1995), for example. Our model is certainly predictive, showing us the regions of cooperation, competition, and the gray area in between. But to have more direct control over which niching situations constitute cooperation and which competition, we need to replace the parameters  $r'_f$  and  $r_{ov}$  with more easily manipulated parameters, such as the probability of crossover  $p_c$  and the selection pressure  $s$  used by

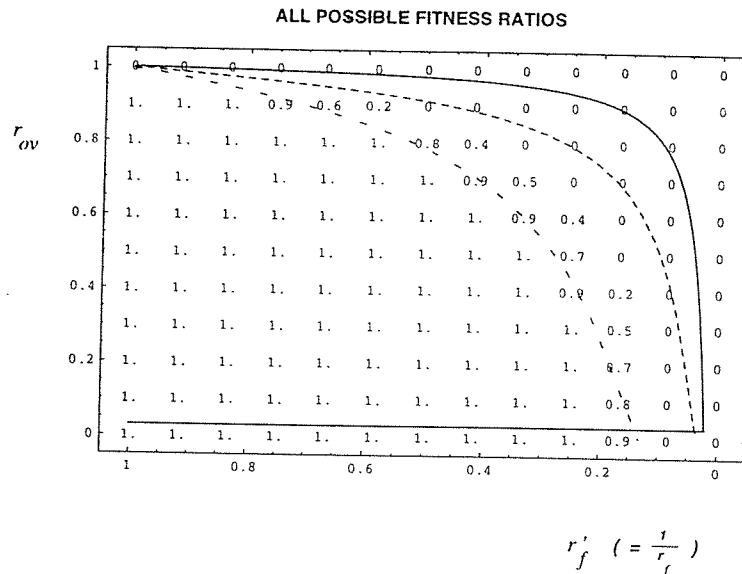


Figure 9: By varying the inverse fitness ratio  $r'_f$ , we can extend our comparison in Figure 8 to include all possible fitness ratios ( $1 \leq r_f < \infty$ ).

Thierens. That is, we want the user of the niched GA to be able to set parameters at the beginning of a GA run, so as to place the boundary between cooperation and competition to ensure the survival of desirable pairs of niches while also ensuring the resolution (convergence) of niche pairs that the user deems competitive.

Such control parameters might not be hard to find. Candidates for fitness sharing control parameters include niche radius  $\sigma_{sh}$ , sharing function exponent  $\alpha_{sh}$ , power sharing exponent  $\beta_{sh}$  (Goldberg, Deb, & Horn, 1992), and population size  $N$ . For example, as  $\beta_{sh}$  increases, the importance of objective (unshared) fitness increases, and the range of "tolerable"  $r_f$  decreases. Similarly, as  $\alpha_{sh}$  and/or  $\sigma_{sh}$  increase, the range of tolerable niche difference (i.e., overlap) decreases. So on the x-axis,  $r_f$  might be replaced by  $\beta_{sh}$ , while on the y-axis  $r_{ov}$  might be replaced by some function of  $\sigma_{sh}$  and  $\alpha_{sh}$ , to yield a more practical control map. This map would allow us to tune our niched GA parameters so that the niche pairs we desire do indeed survive, while the overlapped, conflicting niche pairs we do not want are quickly resolved in favor of the better niche.

For resource sharing, the candidates for control parameters are not as obvious (other than increasing  $N$  to support more overlap and fitness differences). It appears promising however that analogs of fitness sharing's parameters can be found. Metaphors from nature as well as review of common successful techniques in classifier systems and immune system models inspire such optimism. For example, it is not necessary to strictly divide up a resource completely. In nature as well as in some recent LCS work, a resource might become "more available" with increasing demand. Or it might be the case that an individual might not be able to use more than one fraction of a particular resource anyway; thus additional

users of the resource do not so drastically reduce the original user's share. Such effects would allow the niched GA to support increased  $r_{ov}$ . Similarly, if we were to simulate more destructive aspects of competition, such as the severe depletion of a resource with increasing usage, we could increase the importance of  $r_f$  and lower the capacity of the system for handling overlap. Identification of direct control parameters for resource sharing might involve algorithmic details particular to specific implementations, such as antibody-antigen binding strengths as a "niche radius" in immune system models, or the bidding algorithms (auctions) in the LCS. This is clearly a subject for future research.

In general, control over the bounds of cooperation and competition is a worthy goal. For example, it is desirable to reduce the "gray area" between the bounds, since this region constitutes the set of niching situations which are unpredictable. It appears that a simple way of reducing the gray area is to increase population size  $N$ . Larger  $N$  should raise the two bounds plotted in Figure 7, but will likely raise the lower plot (that is, the upper bound on cooperation) at a greater rate than the upper plot. Confirming such an effect would be a short-term extension of the current work.

#### 4.6 OTHER LIMITATIONS

Additional limitations of this predictive map are its restriction to resource sharing and its assumption of a single pair of niches. As for the former limitation, the analysis in this paper can and should be duplicated for fitness sharing, in future work. As for the latter limitation, the map should easily extend to the case of multiple pairs of (possibly overlapping) niches in a single population, with little change. But the extension to three or more tuples of mutually overlapping niches (i.e., triples, quadruples, etc.) is not necessarily straightforward.

### 5 SUMMARY

Increasing fitness difference  $r_f$  and increasing overlap  $r_o$  slow niche convergence time while also speeding up niche loss. We conjecture that when these two characteristic niching times are similar, perhaps differing by an order of magnitude only, niching fails. Applying this conjecture, and solving for the combinations of fitness and overlap that give us these niche failure conditions, we can plot the boundary of niching failure and success. (The boundary turns out to be fairly linear for resource sharing, and indicates that niche loss time dominates the calculation of the cooperation-competition boundary.) This boundary gives us a map predicting which combinations of niche fitness and niche overlap will lead to successful niching (i.e., cooperation) and which will lead to failed niching (i.e., successful competition). Such a navigational tool is the first step in being able to tune our niched GAs to promote exactly the kinds of cooperation and competition we deem appropriate for the problem at hand.

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